Registration of Range Images that Preserves Local Surface Structures and Color

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Abstract

We propose an ICP-based registration method for range images that preserves fundamental features, i.e., local structures and color, of object surfaces. The method employs local surfaces as an attribute for establishing correspondences between range images where local surfaces are evaluated geometrically and photometrically. In estimating correspondences between range images, our method evaluates consistency of shape patterns and chromaticity of local surfaces together. In estimating transformation parameters relating the coordinates between different range images, on the other hand, our method evaluates skewness and chromaticity of correspondences. These two kinds of evaluation enhances accuracy of the estimation and results in preserving local structures and color of object surfaces.

1. Introduction

Registration of range images followed by integration allows us to capture the full geometry of a complicated object and then to generate a 3D model of the object. That is why registration plays an important role in many applications such as CAD or CG. The goal of range image registration is to find the rigid transformation that best aligns given range images.

The iterative closest point (ICP) method proposed by Besl *et al.* [2] is widely used for registration of range images [1, 4, 6, 10, 11, 12, 23]. Basically, it consists of a closest point search and a matching error minimization which are iteratively applied to two range images. To be concrete, the ICP method iterates two steps. Each point in one range image is transformed by a given transformation to find the closest point in the other range image. These point correspondences are then used to estimate the transformation that minimizes matching errors.

Due to digitization depending on viewpoints, the same points on an object surface cannot be measured in different range images even if they are commonly observed. In adAkihiro Sugimoto National Institute of Informatics Tokyo 101-8430, Japan sugimoto@nii.ac.jp

dition, some points on an object surface observed from one viewpoint are not observed from another viewpoint due to self-occlusion. These facts make registration of range images difficult. That is, we have to establish correspondences between range images for registration, providing that true correspondences of measured points do not exist.

To overcome the above problem, the ICP method is extended in different ways. Zhang [26] eliminated false corresponding pairs of points by introducing a threshold for distances of corresponding pairs. Chen *et al.* [3] used only smooth surface parts and minimized the sum of distances between each point in one image and a tangential plane constructed from points in the other image. Masuda [15] introduced a signed distance field to an object surface and matched the fields across range images for registration and integration.

To enhance robustness in searching corresponding features, on the other hand, some other attributes are used in addition to 3D coordinates themselves. They are geometrical attributes [5, 7, 14, 22] computed from 3D coordinates of measured points, intensity attributes [9, 18, 24] and color attributes [8, 13, 17, 21, 25].

Normal vectors with/without curvatures at measured points are evaluated in addition to the 3D coordinates of points [5, 14, 22]. Such differential features, however, are easy to be affected by errors in measuring, and realizing accurate registration using these values themselves is not promising. Godin *et al.* [7] used only the signs of mean curvatures and Gaussian curvatures to reduce the computational cost in searching corresponding points. This method does not pay any attention to the local connectivity of points, which results in not preserving local structures of object surfaces.

In contrast, intensity attributes, in particular, gradients of intensity [18, 24], are introduced to complement geometric attributes. They leads to more sophisticated registration that cannot be attained with only geometric attributes. Color attributes, which provide richer information than intensity, become used for registration recently. Godin *et*

al. [8], Johnson *et al.* [13] and Shütz *et al.* [21] proposed a framework that incorporates color information into the ICP method. They computed an extended distance between measured points which is the weighted sum of norms of the differences between their 3D coordinates or local curvatures and that of RGB values representing their color. RGB values themselves, however, are known to be sensitive to changes in illumination. This indicates that RGB values obtained in different viewpoints are not reliable. In addition, summing attributes in different dimensions is not reasonable. Wyngaerd *et al.* [25] used viewpoint and illumination invariant color information within a local region to search point correspondences but does not pay any attention to consistency of correspondences in any local region.

In this paper, we propose an ICP-based registration method for range images that preserves fundamental features, i.e., local structures and color, of object surfaces. The method employs local surfaces as an attribute for establishing correspondences between range images where local surfaces are evaluated from both geometric and photometric aspects. Our method introduces two geometric features [16] to evaluate local structures of object surfaces: shape patterns of local surfaces and skewness of correspondences. Our method also introduces chromaticity of local surfaces as a photometric feature. Chromaticity eliminates the luminance from color of surfaces and is thus robust against changes in illumination. We make full use of these geometric and photometric features in our registration method. In estimating correspondences across range images, our method evaluates consistency of shape patterns and chromaticity of local surfaces together. In estimating transformation parameters between different range images, on the other hand, our method evaluates skewness and chromaticity of correspondences. These two kinds of evaluation enhances accuracy of the estimation and results in preserving local structures and color of object surfaces.

2. Structural and Color Features for Local Surfaces

2.1. Local surfaces as an attribute

A range image is defined as a set of the 3D coordinates of discretely measured points where the coordinates depend on the viewpoint and its orientation. Let $x^i(u, v)$ denote the coordinates of the measured point that corresponds to the (u, v)-th pixel in the *i*-th range image (u = 1, ..., N;v = 1, ..., M; i = 1, 2). Note that $x^i(u, v)$ depends on the position and the orientation of the viewpoint.

Searching good feature correspondences between range images is crucial in registration. Though measured points across range images do not have true correspondences, local surfaces do have. This is because a range image includes densely measured points and this makes the local connectivity of measured points of an object surface invariant un-



Figure 1. Neighboring points on a local surface and their measured points in range images.

der the change in position and orientation of an viewpoint* (Fig.1). We thus employ local surfaces as an attribute of points for establishing correspondences between range images.

In our method, for each measured point $x^i(u, v)$, we first construct a local surface $S^i(u, v)$ from $x^i(u \pm k, v \pm k)$ (k = -1, 0, 1) in the eight neighboring pixels of (u, v) and attach $S^i(u, v)$ to $x^i(u, v)$ as an attribute. We then use geometric and photometric features of the local surface together to search correspondences across range images. Here we focus on a structural feature from the geometrical point of view whereas focus on chromaticity of surfaces from the photometric point of view.

2.2. Shape patterns

Correct correspondences facilitate accuracy of registration while false correspondences lead to inaccurate registration. How to eliminate false correspondences is thus an important issue.

If a pair of points is correctly corresponding, shape patterns of the attached surfaces, such as convex or concave, are identical with each other. Therefore, among established pairs of correspondences we eliminate the pairs whose shape patterns are different from each other to obtain only the pairs with identical shape patterns.

For each local surface $S^i(u, v)$, we compute the mean curvature $H^i(u, v)$ and the Gaussian curvature $K^i(u, v)$ to identify the shape pattern of $x^i(u, v)$. We use only the signs of these curvatures to classify shape patterns as shown in Table 1. This is because errors incurred in measuring affect curvatures and their values are not expected to be accu-

^{*}The case exists where measured points observed in neighboring pixels are not neighbor on the object surface. In such a case, the measured points are on different surfaces with large difference of distances from the viewpoint due to a special position and orientation of the viewpoint. We do not care about such a special case. In fact, our method does not construct a local surface when the distance between measured points observed in neighboring pixels is large.

Table 1. Shape pattern classification.

	K > 0	K = 0	K < 0
H > 0	convex	convex cylindrical	convex
H = 0	_	planar	saddle
H < 0	concave	concave cylindrical	concave

rate while the signs of these curvatures are robust and reliable. Accuracy in establishing corresponding pairs is thus enhanced.

Employing only the corresponding pairs of points with identical shape patterns preserves local surface structures during registration processes. We remark that we only have to construct local surfaces and classify their shape patterns once and for all in advance. This is because the construction of local surfaces and the classification into shape patterns are both independent of correspondences and transformations between range images.

2.3. Chromaticity

From the geometrical point of view, corresponding pairs of points with identical shape patterns contribute to accurate and robust registration. From the photometric point of view, however, they are not sufficient because a pair of points with identical shape patterns exists where their color is completely different from each other. We thus have to guarantee consistency of not only structural features but also color features to establish good correspondences.

We cannot evaluate color itself because color is sensitive to illumination conditions. In capturing range images of an object, we change viewpoints relative to the object, which may cause great changes in illumination of a local surface of the object. This indicates that evaluating consistency of color directly using RGB values is not effective. We, therefore, eliminate the luminance from color information, and then use chromaticity of a local surface to evaluate consistency of color. Chromaticity of a point is defined as a 2D vector that is obtained by projecting the intersection of the RGB vector representing color of the point and the unit plane[†] onto the RG plane.

Since points on an object surface are densely measured in range images, a constructed local surface can be assumed to have uniform color. We thus compute the average of chromaticity of points over the surface to obtain chromaticity of the local surface. This averaging is expected to stabilize chromaticity of the local surface. We note that computing chromaticity of local surfaces should be conducted once and for all in advance.



Figure 3. Correspondence vectors in a local surface.

3. Skewness of Correspondences

For accurate and robust estimation of transformation parameters, a function evaluating transformation parameters should have the minimum when true parameters relating two range images are given, and it should not have local minimums around the true parameters. Therefore, to reduce local minimums of an evaluation function and, at the same time, to preserve structures of local surfaces, our method introduces skewness of correspondences [16]. The skewness of correspondences is a criterion that evaluates how the end point of a vector is affected by the displacement of the staring point of the vector. With the skewness of correspondences, we can evaluate rigidity of correspondences around neighboring points.

Let \mathcal{T} denote the transformation parameters that transform the coordinates of the first range image to those of the second. We assume here that correspondences of points between the first and second range images are given. For each corresponding pair of points $x^1(u, v)$ and $x^2(u', v')$, we define a *correspondence vector* whose starting point is $\mathcal{T}(x^1(u, v))$ and whose end point is $x^2(u', v')$, as shown in Fig.2. We then evaluate consistency of the correspondence vectors obtained in neighboring pixels of (u, v).

As a range image includes densely measured points, directions and norms of the correspondence vectors obtained in neighboring pixels become uniform if transformation parameters are correctly estimated. For example, Fig.3 shows two cases in transformation estimation where two range images are sufficient close to each other and norms of correspondence vectors are almost the same. (a) in Fig.3 is the case where directions of correspondence vectors are uniform whereas (b) in Fig.3 is the case where the directions



 $^{^\}dagger \text{The}$ unit plane is defined as the plane going through the three points (1,0,0),(0,1,0) and (0,0,1) in the RGB space.

are scattered. We see that the transformation parameters for (a) are preferred rather than those for (b). Skewness of correspondences discriminates (a) from (b) with giving a higher score to (a) and thus leads to reduction in local minimums of the evaluation function. We note that distance of correspondences alone does not discriminates (a) from (b).

Skewness $s(T(x^1(u, v)), x^2(u', v'))$ is defined as the sum of eigenvalues of the skew tensor [20] which is computed with the correspondence vectors in the eight neighboring pixels of (u, v) in the first range image. This value becomes smaller as correspondence vectors become more uniform. We note that if transformation parameters change, correspondence vectors also change, which results in changes in skewness. Skewness should be thus computed whenever transformation parameters change.

4. Registration with Geometrical and Photometric Criteria

Our registration method iterates two steps as in the ICP method. One is searching corresponding points and the other is estimating transformation parameters. How to evaluate the distance between correspondences is an important issue because the distance plays a central role in each of the two steps. We introduce evaluation from the geometrical point of view and the photometric points of view together.

At the beginning of registration, for each point $x^i(u, v)$, we identify the shape pattern and compute chromaticity $c^i(u, v)$ using $S^i(u, v)$. Because shape patterns and chromaticity remain invariant under the change in transformation, their classification and calculation are conducted only once at the beginning of registration.

4.1. Searching corresponding pairs

In searching corresponding points, we employ shape patterns as a geometric feature and chromaticity as a photometric feature. In addition, to avoid summing features in different dimensions, we use chromaticity as a weight in computing geometric distances between points. Geometric distance weighted by chromaticity gives us a criterion measuring geometrical and photometric consistency of corresponding points.

For given transformation parameters \mathcal{T} , corresponding points are searched under the criterion of the geometric distance weighted by chromaticity. We then establish tentative corresponding points.

Next, we check shape patterns of all the tentative corresponding points, and then eliminate the pairs whose shape patterns are not identical. We also check chromaticity between the tentative corresponding pairs, and eliminate the pairs whose chromaticity is different from each other.

As a result, we obtain only the corresponding pairs of points with consistent geometric structures and color. We call them checked corresponding pairs of points to discriminate them from tentative corresponding pairs. **Distance between points weighted by chromaticity.** To evaluate the distance between points based not only on the geometrical viewpoint but also on the photometric viewpoint, we introduce a geometric distance weighted by chromaticity.

Let $x^1(u, v)$ and $x^2(\tilde{u}, \tilde{v})$ be a tentative corresponding pair of points. The distance of $x^1(u, v)$ and $x^2(\tilde{u}, \tilde{v})$ is then defined by

$$d_w(\mathcal{T}(\boldsymbol{x}^1(u,v)), \boldsymbol{x}^2(\tilde{u},\tilde{v}))) = |\boldsymbol{c}^1(u,v) - \boldsymbol{c}^2(\tilde{u},\tilde{v})| \cdot d(\mathcal{T}(\boldsymbol{x}^1(u,v)), \boldsymbol{x}^2(\tilde{u},\tilde{v})), (1))$$

where $|c^1(u,v) - c^2(\tilde{u},\tilde{v})|$ is the norm of $c^1(u,v) - c^2(\tilde{u},\tilde{v})$, and $d(\mathcal{T}(\boldsymbol{x}^1(u,v)), \boldsymbol{x}^2(\tilde{u},\tilde{v}))$ is the Euclidean distance between $\mathcal{T}(\boldsymbol{x}^1(u,v))$ and $\mathcal{S}^2(\tilde{u},\tilde{v})$. We remark that in computing the Euclidean distance of a corresponding pair of points $\mathcal{T}(\boldsymbol{x}^1(u,v))$ and $\boldsymbol{x}^2(\tilde{u},\tilde{v})$, we use the Euclidean distance from $\mathcal{T}(\boldsymbol{x}^1(u,v))$ to the local surface attached to $\boldsymbol{x}^2(\tilde{u},\tilde{v})$. This is because no true point correspondences exist across range images.

4.2. Estimating transformation parameters

For given checked corresponding pairs of points, we estimate the transformation parameters that minimize distances between the checked corresponding pairs and, at the same time, preserves consistency of the correspondences where the distances are weighted by chromaticity and consistency is evaluated in terms of skewness. We note that this estimation also involves evaluation from the geometrical aspect and the photometric aspect together.

To evaluate transformation parameters \mathcal{T} , we define the following function $J(\mathcal{T})$:

$$J(\mathcal{T}) = \alpha J_d + (1 - \alpha) J_s.$$
⁽²⁾

where J_d is the distance term weighted by chromaticity, J_s is the skewness term, and α is a weighting function between J_d and J_s .

To reduce the influence of false correspondences not eliminated in the step of searching corresponding pairs, we employ ρ function which is commonly used in Mestimator [19]. Accordingly, J_d and J_s are expressed by

$$J_d = \sum_{u,v} \rho[d_w(\mathcal{T}(\boldsymbol{x}^1(u,v)), \boldsymbol{x}^2(u',v')), d_\gamma],$$

$$J_s = \sum_{u,v} \rho[s(\mathcal{T}(\boldsymbol{x}^1(u,v)), \boldsymbol{x}^2(u',v')), s_\gamma],$$

where d_{γ} and s_{γ} are thresholds for the geometric distance weighted by chromaticity and skewness, respectively, and

$$\rho[t,\gamma] = \frac{t^2}{(t^2 + \gamma)}$$

Note that $x^1(u, v)$ and $x^2(u', v')$ is a checked corresponding pair of points.



If corresponding pairs of points are fixed, differences of chromaticity between the corresponding pairs do not depend on transformation parameters. As a result, $|c^1(u,v) - c^2(u',v')|$ does not change and it can be factored out as a constant in the computation. Note that $d(\mathcal{T}(\boldsymbol{x}^1(u,v)), \boldsymbol{x}^2(u',v'))$ in (1) itself does change while transformation parameters change even for fixed corresponding pairs of points.

Weighting function between J_d and J_s . We dynamically determine the weighting function α using the coefficient of variation of distances between checked corresponding pairs of points. That is, we dynamically determine α_k in the k-th iteration by

$$\alpha_k = \frac{1}{2} \frac{\sigma_k}{m_k} / \frac{\sigma_0}{m_0},$$

where m_k and σ_k are respectively the mean and the standard deviation of distances $d_w(\mathcal{T}(\mathbf{x}^1(u, v)), \mathbf{x}^2(u', v'))$ over the checked corresponding pairs of points in the k-th iteration, and m_0 and σ_0 are those for the initial transformation parameters. This is based on the following observations.

At the beginning of registration, corresponding pairs of points may not be so close to each other and we have a wide variety of weighted distances between checked corresponding pairs. The distance weighted by chromaticity should thus play an more important role than skewness. After several iterations, on the other hand, weighted distances between checked corresponding pairs are expected to become small enough and we have almost uniform weighted distances. Then, to preserve local surface structures and to reduce local minimums, the skewness gradually become more important.

This dynamic control of the weighting function α facilitates reduction of the number of iterations required for registration.

5. Description of Algorithm

Based on the discussion above, we present here our algorithm for registration of range images. Our algorithm is based on the framework of the ICP method.

- **Step 1:** For each pixel (u, v) in each range image i (i = 1, 2), do the following procedures.
 - (a) Construct local surface $S^i(u, v)$ from the measured points $x^i(u \pm k, v \pm k)(k = -1, 0, 1)$ that are observed in the eight neighboring pixels in range image *i* and store it as an attribute of $x^i(u, v)$.
 - (b) Identify the shape pattern of $S^i(u, v)$.
 - (c) Compute chromaticity $c^i(u, v)$.
- Step 2: Set initial values for transformation parameters.



Figure 4. Synthetic range images with color (top view).

- **Step 3:** Iterate the two steps below until the value of evaluation function J in (2) converges. If (2) converges, then go to Step 4.
 - (a) i. For each point in the first range image, search the closest point in the second range image under the criterion of (1) to obtain tentative corresponding pairs of points.
 - ii. Check consistency of shape patterns and chromaticity of the tentative corresponding pairs to obtain checked corresponding pairs of points.
 - (b) Estimate the transformation parameters that minimize (2) using the checked corresponding points obtained in Step 3(a)ii.
- **Step 4:** Transform the coordinates of all the points in the first range image to the second range image by using the estimated transformation parameters, and align the two range images.

6. Experiments

To demonstrate the potential applicability of the proposed method, we applied the method to synthetic range images and real range image. In these experiments, we focused on observing the effectiveness of evaluating geometric and photometric aspects together. As for the effectiveness of introducing shape patterns and skewness, see [16].

6.1. Registration using synthetic range images

We generated a situation where we capture two range images of a rotationally symmetric solid object with color. This indicates that we cannot obtain successful registration of the range images without color information. To generate a synthetic range image of 20×20 points with color, we first set a 3D coordinate system and regarded its origin as the viewpoint. We then generated the 3D coordinates of 20×20 points according to

$$z = -\left\{\cos\left(\frac{\pi}{100}x^2\right) + \frac{\pi}{100}y^2\right\},$$

where x and y are uniformly sampled from [-2, 14] and [-5, 9], respectively. We remark that the shape of an object





Figure 5. Synthetic range images from different viewpoints.





(b) after registration Figure 6. Registration results.

is like a vase or a saddle and it is rotationally symmetric with respect to the x-axis. Next, we segmented points into 4 regions based on their x and y coordinates, to each of which we attached one set of RGB values to obtain a range image with color. We also rotated all the points by 22.5° with respect to the z-axis to obtain another range image with color. Finally, we perturbed z-coordinate of each point in the first and second images by independently adding Gaussian noise with the mean of 0.0 and the standard deviation of 0.01. In this way, we prepared two range images with color. They are shown in Figs. 4 and 5. Fig. 4 shows color information attached to the points in the two range images. Fig. 5 clearly show how points in the images are distributed in 3D where the viewpoint was selected just for this presentation.

To these two range images, we applied our registration method, the result of which is shown in Fig.6. Note that two viewpoints were selected again for a clear presentation. We



Figure 7. Behavior of J around the true transformation parameters (darkness means smallness; the *z*-axis is horizontal).

observe that our method realizes successful registration of the two range images. We see that combining geometric and photometric features together in all aspects of registration leads to our successful result.

To confirm the effectiveness of incorporating geometric and photometric aspects together, we compared our method with the method ignoring color. In both the methods, checking consistency of shape patterns and chromaticity are used for establishing correspondences (the same point correspondences are thus used for the evaluation of transformation parameters), and skew of correspondences is also evaluated.

Behaviors of J, which is the function to evaluate transformation parameters, around the true transformation parameters are shown in Fig.7 in terms of level curves. While the transformation has 6 parameters, Fig.7 shows the values of J only with respect to the angle around the x-axis and the displacement along the z-axis. (a) is for the proposed method, and (b) is for the method ignoring color. We observe that ambiguity remains along the angle around the x-axis in (b) while such ambiguity does not exist in (a). This observation verifies that our J correctly identifies the true transformation parameters.

6.2. Registration using real range images

We evaluated our registration method using real range images. We employed the PS-3300C from LDI as a range sensor and obtained two range images of a vessel (Fig.8) from two different viewpoints, which are shown in Fig.9. We selected about 3000 points from each range image (about 20000 points) for our registration. The angle between two viewing directions was about 15 degrees. Note that this vessel is rotationally symmetric and that these range images cannot be successfully aligned without color information.

We first aligned the two range images with manually selected initial transformation parameters (Fig.10). We then applied our method to these two images. To confirm the effectiveness of combining the geometrical and photometrical aspects in evaluation, we compared our method with a method ignoring color. The other method is different from ours in the only sense that it ignores chromaticity of lo-



Figure 8. The vessel used in the experiment.



Figure 9. Two range images of the vessel.

cal surfaces. The registration results of our method and the other method are shown in Fig.11 and Fig.12, respectively. In these figures, the left ones are presented in the front view and the right ones are presented in the bottom view to show how points in the images are distributed in 3D.

We see a gap between two range images in Fig.10. In particular, it appears remarkably around the brim of the vessel, marked with a blue square. The both methods correctly removed the gap in the geometrical sense. In the photometrical sense, on the other hand, our method is satisfactory while the other method is not. The boundary area between yellow and brown surfaces, marked with a white square in Fig.8, is correctly aligned by our method as seen in Fig.11, while the other method failed (Fig.12). We may thus conclude that our method realizes successful registration not only geometrically but photometrically.

Fig.13 shows the evaluation function J depending on iterations, comparing our method with the other method. Note that their evaluation functions are different from each other. Both the methods actually converge at a global minimum. Our method converges with about 20 iterations, while the other method with 8 iterations. The reason why our method requires more iterations may be that the initial transformation parameters used in the experiment had an advantage in geometrical registration.

7. Conclusion

We propose an ICP-based registration method for range images that incorporates local connectivities and color into both evaluation on correspondences and evaluation on transformation parameters. The method employs local surfaces as an attribute for establishing correspondences between range images where local surfaces are evaluated from both geometric and photometric aspects. To preserve local



Figure 10. Before registration.



Figure 11. After registraion by our method.



Figure 12. After registration by the method ignoring color.

structures of object surfaces, our method introduces two geometric features: shape patterns of local surfaces and skewness of correspondences. To preserve consistency of color between corresponding points, it also introduces chromaticity of local surfaces as a photometric feature. Chromaticity eliminates the luminance from color of the surfaces and provides robustness against changes in illumination.

We make full use of the introduced geometric and photometric features in our registration method from the geometrical point of view and also from the photometric point of view. In estimating correspondences between range images, our method evaluates consistency of shape patterns and chromaticity of local surfaces together. In estimating transformation parameters, on the other hand, our method evaluates skewness and chromaticity of correspondences. These two kinds of evaluation enhances accuracy and robustness of estimation and results in preserving fundamental features, i.e., local structures and color, of object sur-





Figure 13. The value of J in the estimation depending on iterations for the real range images.

faces.

Estimated transformation parameters can be used to predict changes in luminance between two range images. Incorporating these changes into evaluation on corresponding pairs of points will lead to more accurate registration. An extension of our method into this direction is left for future work.

Acknowledgements This work is in part supported by Grant-in-Aid for Scientific Research of the Ministry of Education, Culture, Sports, Science and Technology of Japan under the contract of 13224051, 14380161 and 16650040.

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