

# Range Image Registration Preserving Local Structures of Object Surfaces

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## Abstract

We propose a registration method for range images that preserves local structures of object surfaces. The method introduces shape patterns and a skewness of correspondences, both of which are extracted from the local surface nearby a point of interest in each image. The shape patterns are used to eliminate false corresponding pairs of surfaces, while the skewness is used to estimate the transformation that relates the coordinates between different range images. These two features enable us to estimate the transformation that preserves local structures of object surfaces.

## 1. Introduction

Automatic 3D model acquisition of the real world object is important for many applications in CAD/CAM, CG, etc. For such 3D model acquisition, a number of methods using range images have been proposed [6, 9, 11, 13].

A range image includes a partial shape of an object in terms of an individual 3D coordinate system that depends on the viewpoint of a range sensor. Therefore, to obtain the total shape of the object, the range image registration, i.e., the estimation of transformations between coordinate systems, is indispensable.

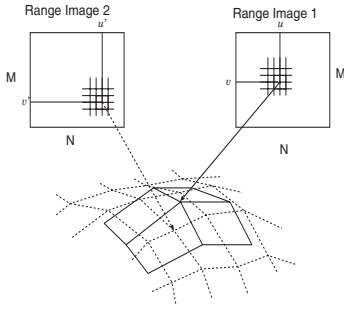
The standard approaches to the range image registration take two steps. First, correspondences of some extracted features or the coordinates themselves are established across range images. The transformation parameters, i.e., the position and the orientation of the viewpoint, are then estimated based on the established correspondences.

The range image registration involves difficulty in the following sense. A range image is given as a set of discretely measured points and different range images obtained from different viewpoints do not usually include the same measured points. That is, we have to establish correspondence between range images under the condition that true correspondences of points do not exist. In addition, some points of an object surface observed from one viewpoint are not observed from another viewpoint due to occlusion. This means that we have no way to know commonly observed points until the registration is successfully finished. However, we cannot proceed to the registration without establishing correspondences of commonly observed points across range images.

On the other hand, the iterative closest point (ICP) method was proposed by Besl *et al.* [1] within the context of the pose estimation of a range image using a 3D object model. The ICP method iterates two steps. The first step establishes point correspondences between a range image and a model based on a given transformation. That is, each point in the range image is transformed by the given transformation parameters to find the nearest point in the model as its correspondence. The second step estimates transformation parameters based on the established correspondences. In this step, the transformation parameters minimizing distances between the corresponding pairs of points are obtained.

The ICP method was extended to the range image registration. To avoid the problem that true correspondences do not exist in the range image registration, Zhang [17] eliminated false corresponding pairs of points by introducing a threshold for distances of corresponding pairs. Chen *et al.* [2] used only smooth surface parts and minimized the sum of distances between each point in one image and a tangential plane constructed from points in the other image. Subsequently proposed methods [3, 8, 16] are also extensions of the ICP method to improve the accuracy of the registration. Some [4, 10, 15] used normal vectors and/or curvatures at measured points in addition to their coordinates. Because such differential features are easy to be influenced by the measurement error, realizing the accurate registration using these values themselves is difficult. Instead of differential features themselves, Godin *et al.* [7] used only the signs of mean curvatures and Gaussian curvatures to reduce the computational cost in searching corresponding points. However, their method does not preserve local structures of object surfaces. This is because their method does not pay any attention to the local connectivity of points.

We propose a registration method that evaluates surface structures to preserve local shapes of an object. We employ local surfaces for establishing correspondences between range images, and then introduce two novel features to evaluate local surface structures. They are *shape patterns of local surfaces* and *a skewness of correspondences*. The first one is used to eliminate false correspondences. The second one is used to evaluate consistency of the correspondences in neighboring pixels. Evaluating these two features leads to the accurate and robust registration of range images.



**Figure 1. Neighbouring points on a local surface and their measured points in range images.**

## 2. Structural Features of Local Surface

Let us denote by  $\mathbf{x}^i(u, v)$  ( $u = 1, \dots, N; v = 1, \dots, M; i = 1, 2$ ) the coordinates of a point measured in the  $(u, v)$ -th pixel of the  $i$ -th range image. Note that  $\mathbf{x}^i(u, v)$  depends on the position and the orientation of the viewpoint where the  $i$ -th range image is obtained.

### 2.1. Shape patterns of local surfaces

Obtaining good feature correspondences between range images is crucial. Since true correspondences of measured points across range images do not exist, we employ local surfaces instead of points for establishing correspondences between range images. This is because the local connectivity of measured points of an object surface remains invariant under the change in position and orientation of an viewpoint\* (Fig.1).

For each measured point  $\mathbf{x}^i(u, v)$ , we construct a local surface  $\mathcal{S}^i(u, v)$  from points  $\mathbf{x}^i(u \pm k, v \pm k)$  ( $k = -1, 0, 1$ ), all of which are measured in the eight neighboring pixels of  $(u, v)$  in the range image. Then, we establish corresponding pairs of local surfaces.

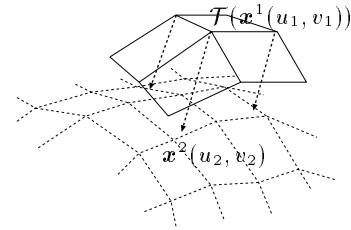
In searching corresponding pairs of local surfaces, how to eliminate false correspondences is an important issue for the accurate registration. If a pair of local surfaces is not correctly corresponding, the shape of the surfaces, such as convex or concave, is not identical with each other. Therefore, we use shape information of surfaces to eliminate false correspondences.

To obtain shape information of local surface  $\mathcal{S}^i(u, v)$ , we first compute the mean curvature  $H^i(u, v)$  and the Gaussian curvature  $K^i(u, v)$  [5]. We then use only the sign of these curvatures to classify the shape of  $\mathcal{S}^i(u, v)$  into six patterns, as shown in Table 1. This is because the signs of these curvatures are robustly computed, even though the curvatures themselves are not accurately computed due to measurement errors.

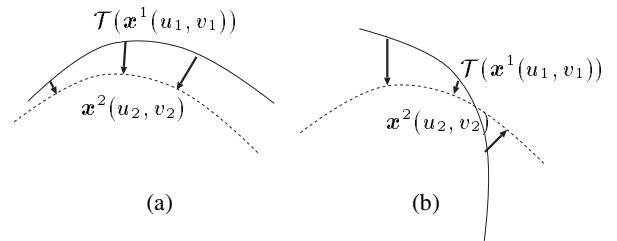
\*The case exists where measured points observed in the neighboring pixels are not neighbor on the object surface. In such a case, the measured points are on different surfaces with large difference of distances from the viewpoint due to a special position and orientation of the viewpoint. We do not care about such a special case. In fact, our method does not construct a local surface when the distance between measured points observed in the neighboring pixels is large.

**Table 1. The classification of shape patterns.**

	$K > 0$	$K = 0$	$K < 0$
$H > 0$	convex	convex cylindrical	convex
$H = 0$	-	planar	saddle
$H < 0$	concave	concave cylindrical	concave



**Figure 2. The correspondence vector.**



**Figure 3. The skewness of correspondences.**

In this way, accuracy in establishing corresponding pairs is enhanced. Using only the corresponding pairs of local surfaces with identical shape patterns preserves local surface structures in the registration.

### 2.2. Skewness of corresponding surfaces

Transformation parameters  $\mathcal{T}$ , which transform the coordinates of the first range image to that of the second, are estimated using corresponding pairs of local surfaces between range images. For the accurate and robust estimation of transformation parameters regardless of an initial estimation, a function evaluating transformation parameters should have the minimum when the true parameters are given, and it should not have local minimums around the true parameters.

To reduce local minimums of an evaluation function, our method introduces the skewness of correspondences that evaluates consistency of the correspondences within neighboring points.

For each corresponding pairs of local surfaces  $\mathcal{S}^1(u_1, v_1)$  and  $\mathcal{S}^2(u_2, v_2)$ , the *correspondence vector* is defined as the vector whose starting point is  $\mathcal{T}(\mathbf{x}^1(u_1, v_1))$  and end point is  $\mathbf{x}^2(u_2, v_2)$ , as shown in Fig.2. We evaluate consistency of the correspondence vectors obtained in the eight neighboring pixels of  $(u_1, v_1)$ .

The skewness  $s(\mathcal{T}(\mathcal{S}^1(u_1, v_1)), \mathcal{S}^2(u_2, v_2))$  of the corresponding pair of local surfaces  $\mathcal{S}^1(u_1, v_1)$  and  $\mathcal{S}^2(u_2, v_2)$  is the sum of eigenvalues of the skew tensor<sup>†</sup> determined

<sup>†</sup>Let  $\mathbf{v} = (v_x, v_y, v_z)^\top$  be a correspondence vector. Then the skew

by correspondence vectors in the eight neighboring pixels. This value expresses the amount of displacements of the end points caused by that of the starting points. In other words, the skewness expresses rigidity of correspondences around the neighboring points because it becomes smaller when the correspondence vectors become more uniform.

If transformation parameters are correct, for example, the directions and the norms of correspondence vectors should be uniform around the neighboring points. Therefore, the transformation parameters that give uniform correspondence vectors around the neighboring points are preferred rather than those that give scattered correspondence vectors, as shown in Fig.3. Evaluating only the norms of corresponding vectors cannot discriminate (a) and (b) in Fig.3. In contrast to this, evaluating the skewness of correspondences does discriminate (a) from (b). This leads to reduction in local minimums of the evaluating function.

### 3. Registration Using Local Features of Surface Structures

For the registration of two range images, we employ alternative iterations of two steps, as in the ICP method. One is establishing correspondences of local surfaces, and the other is estimating the transformation. The shape patterns and the skewness play important roles in our method. The shape patterns are used in the correspondence estimation. The skewness is used in the transformation estimation.

At the beginning of the registration, for each point  $x^i(u, v)$ , the local surface  $\mathcal{S}^i(u, v)$  is constructed and its shape pattern is classified (cf. Table 1). Because shape patterns remain invariant under the change in transformation, their classification is done only once at the beginning of the registration.

#### 3.1. Finding corresponding pairs

For given transformation parameters  $\mathcal{T}$ , corresponding pairs of local surfaces are established based on the distance and consistency of shape patterns.

First, tentative corresponding pairs of the local surfaces are established based on the distance. To be concrete, for each transformed local surface in the first range image, we select the nearest local surface among all the local surfaces in the second range image. We then check shape patterns of all the tentative corresponding pairs, and eliminate the pairs whose shape patterns are not identical. As a result, we obtain only the corresponding pairs having the same local structures.

**The distances between local surfaces.** For the simplicity in the computation of the distance between local surfaces,

tensor [14] is given by

$$\begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{1}{2} \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) & \frac{1}{2} \left( \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) \\ \frac{1}{2} \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right) & \frac{\partial v_y}{\partial y} & \frac{1}{2} \left( \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right) \\ \frac{1}{2} \left( \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right) & \frac{1}{2} \left( \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} \right) & \frac{\partial v_z}{\partial z} \end{bmatrix}.$$

we use the distance from a point in the first range image to the closest triangular patch constructed from the points in the second range image. For example, for a local surface  $\mathcal{S}^1(u_1, v_1)$ , its tentative corresponding local surface and the distance is computed as follows. We first select the triangular patch with the shortest distance from  $\mathcal{T}(\mathcal{S}^1(u_1, v_1))$  among all the triangular patches in the second image. Let  $d_1$  be the distance. We then select a representative point of this patch. Let  $x^2(u'_2, v'_2)$  be the point. We regard that  $\mathcal{S}^2(u'_2, v'_2)$  is the local surface corresponding to  $\mathcal{S}^1(u_1, v_1)$  and  $d_1$  is the distance between the two surfaces.

#### 3.2. Estimating transformation

For given corresponding pairs of local surfaces, we evaluate the distances and consistency of the correspondences to estimate transformation parameters  $\mathcal{T}$ .

The evaluation function  $J(\mathcal{T})$  is expressed by

$$J(\mathcal{T}) = (1 - \alpha)J_s + \alpha J_d, \quad (1)$$

where  $J_s$  is the skewness term,  $J_d$  is the distance term, and  $\alpha$  is the weighting function between  $J_s$  and  $J_d$ .

To reduce the influence of false correspondences not eliminated in the shape pattern check, we employ  $\rho$  function that is commonly used in M-estimator [12]. Namely,  $J_s$  and  $J_d$  are concretely expressed by

$$J_s = \sum_{u_1, v_1} \rho[s(\mathcal{T}(\mathcal{S}^1(u_1, v_1)), \mathcal{S}^2(u_2, v_2)), s_\gamma],$$

$$J_d = \sum_{u_1, v_1} \rho[d(\mathcal{T}(\mathcal{S}^1(u_1, v_1)), \mathcal{S}^2(u_2, v_2)), d_\gamma],$$

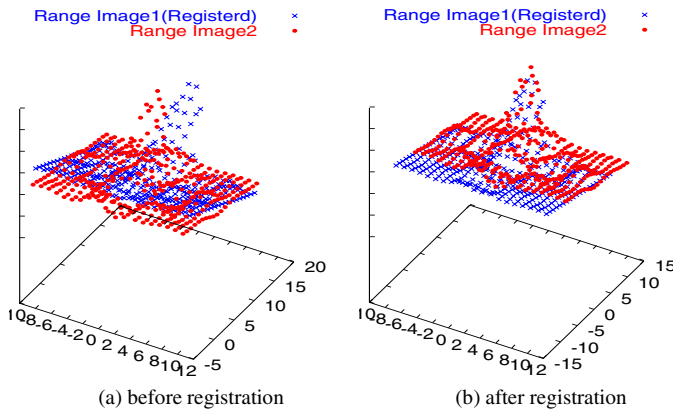
where  $s_\gamma$  and  $d_\gamma$  are thresholds for the skewness and the distance, and  $\rho[t, \gamma] = \frac{t^2}{(t^2 + \gamma)}$ . Note that the established corresponding pairs of local surfaces are  $\mathcal{S}^1(u_1, v_1)$  and  $\mathcal{S}^2(u_2, v_2)$ .

The distance  $d(\mathcal{T}(\mathcal{S}^1(u_1, v_1)), \mathcal{S}^2(u_2, v_2))$  is computed as is in the step of finding corresponding pairs.

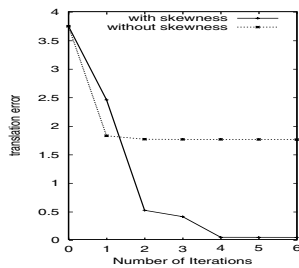
**Weighting function between  $J_s$  and  $J_d$ .** We dynamically determine the weighting function  $\alpha$  using the coefficient of variation with respect to distances between corresponding surfaces. That is, we dynamically determine  $\alpha$  by  $\alpha = \frac{1}{2} \frac{\sigma_k}{m_k} / \frac{\sigma_0}{m_0}$ , where  $m_k$  and  $\sigma_k$  are the mean and the standard deviation of  $d(\mathcal{T}(\mathcal{S}^1(u_1, v_1)), \mathcal{S}^2(u_2, v_2))$  at the  $k$ -th iteration, respectively, and  $m_0$  and  $\sigma_0$  are those for the initial transformation parameters. This is based on the following observations.

At the beginning of the registration, corresponding pairs are not so close to each other. The distance term thus plays an important role. After several iterations, on the other hand, distances between corresponding pairs are expected to be short enough. Then, to preserve local surface structures, the skewness term becomes important.

This dynamic control of  $\alpha$  facilitates reduction in the number of iterations required for the registration.



**Figure 4. The registered result using synthetic range images.**



**Figure 5. Translation errors in the estimation depending on iterations.**

#### 4. Experimental Results

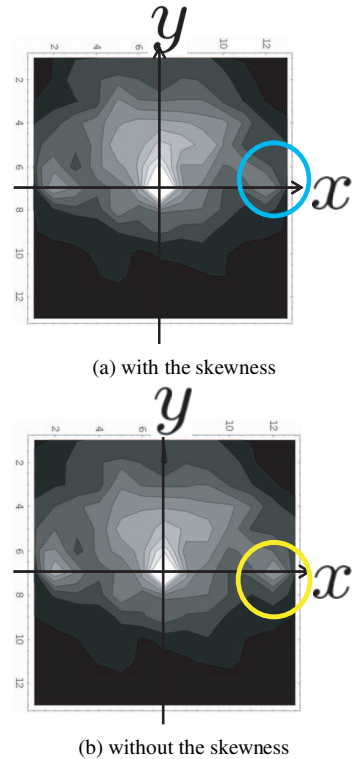
We generated two synthetic range images (Fig.4(a)). The image size was  $20 \times 20$ , and the angle between two viewing directions was 20 degrees. To these two range images, we applied our registration method, the result of which is shown in Fig.4(b).

We observe that our method realizes the successful registration of the two range images. The rate of corresponding pairs eliminated due to different shape patterns was 18%. This verifies the effectiveness of introducing shape patterns in establishing corresponding pairs.

To confirm the effectiveness of the skewness, we compared our method with the method without the skewness where  $\alpha = 1$  in the evaluation function. The results are shown in Fig.5 and Fig.6. In the both methods, shape patterns were used to establish correspondences.

Figure 5 shows errors of the estimated translation vectors with respect to iterations. We see that our method converges with a small number of iterations. While the translation error of the method without the skewness is large even at the converged parameters, the translation error of our method is very small. This indicates that the method without the skewness was trapped by a local minimum.

Behaviors of  $J$  around the true transformation, on the other hand, are shown in Fig.6 in the forms of level curves. Though the transformation has 6 parameters, Fig.6 shows



**Figure 6. Behaviors of  $J$  around the true transformation (brightness means smallness).**

the values of  $J$  only with respect to the displacement of  $x$  and  $y$  coordinates. (a) is for the proposed method, and (b) is for the evaluation function with only the distance term. We observe that local minimums are reduced in the evaluation function of (a) in comparison with that of (b). In particular, we see that a local minimum of (b) along the  $x$  axis, marked by a circle, was removed in (a).

We also applied our registration method to real range images. We employed the PS-3300C from LDI as the range sensor and obtained two range images of a doll (Fig.7) from two different viewpoints. The number of points in each range image used in this experiment was about 2000. We then applied our method to these two images. The results are shown in Fig.8 from two different viewpoints selected just for this presentation to show how points in the images are distributed in 3D. The points in the first and the second range images are expressed in red and blue, respectively. We see that our method is also valid for real range images. The rate of corresponding pairs eliminated due to different shape patterns was 24% in this case.

Figure 9 shows the values of  $J$  with respect to iterations for the registration of the real range images. We see that our method converges with only about 10 iterations. This fast convergence is realized because of our dynamic control of weighting function  $\alpha$ .



Figure 7. The doll used in the experiment.

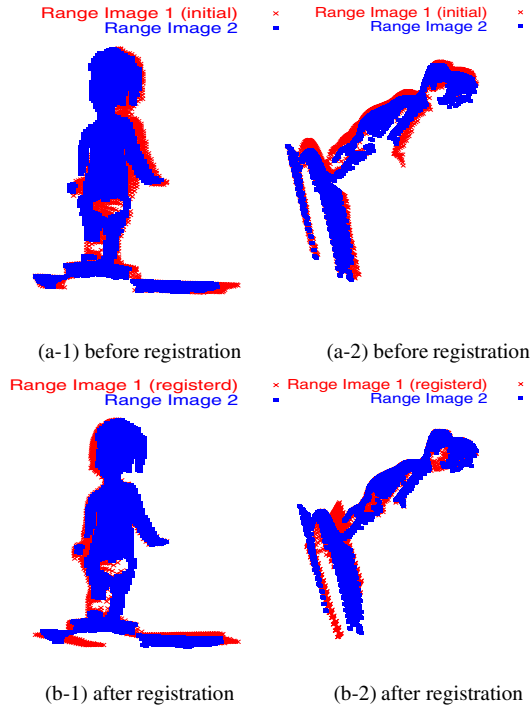


Figure 8. The registered result using real range images!

## 5. Conclusion

We proposed a registration method of two range images that incorporates local connectivities and consistency of correspondences. We employed local surfaces for establishing correspondences between range images. To preserve local shapes of object surfaces, we introduced two local features: shape patterns and the skewness of correspondences. The shape patterns, extracted from measured points in neighboring pixels in each image, were used to eliminate false correspondences. The skewness of correspondences was used to evaluate consistency of the corresponding pairs in the neighboring pixels. With these two novel features, the accurate and robust registration of range images was realized. Our experimental results showed the effectiveness of our method.

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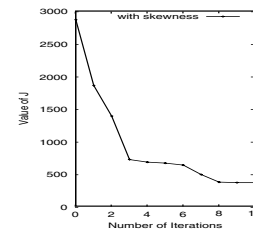


Figure 9. The value of  $J$  in the estimation depending on iterations for the real range images.

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## References

- [1] P. J. Besl and N. D. McKay. A Method for Registration 3-D Shapes. *IEEE Trans. on PAMI*, 14(2):239–256, 1992.
- [2] Y. Chen and G. Medioni. Object Modeling by Registration of Multiple Range Images. *IVC*, 10(3):145–155, 1992.
- [3] C. Dorai, G. Wang, A. K. Jain, and C. Mercer. Registration and Integration of Multiple Object Views for 3D Model Construction. *IEEE Trans. on PAMI*, 20(1):83–89, 1998.
- [4] J. Feldmar, N. Ayache, and F. Berrig. Rigid, Affine and Locally Affine Registration of Free-Form Surfaces. *IJCV*, 18(2):99–119, 1996.
- [5] D. Forsyth and J. Ponce. *Computer Vision – A Modern Approach*. Prentice Hall, 2003.
- [6] N. Gelfand, S. Rusinkiewicz, L. Ikemoto, and M. Levoy. Geometrically Stable Sampling for the ICP Algorithm. *Proc. of 3DIM*, pp. 260–267, 2003.
- [7] G. Godin and P. Boulanger. Range Image Registration Through Invariant Computation of Curvature. *Proc. of ISPRS Workshop From Pixels to Sequences*, pp. 170–175, 1995.
- [8] E. Guest, E. Berry, R. A. Baldock, M. Fidrich, and M. A. Smith. Robust Point Correspondence Applied to Two and Three-Dimensional Image Registration. *IEEE Trans. on PAMI*, 23(2):165–179, 2001.
- [9] K. Ikeuchi. Overview of CREST Digital Archiving Project – Digital Archiving of Cultural Heritage through Observation. *Proc. of International Symposium on the CREST Digital Archiving Project*, pp. 3–15, 2003.
- [10] P. Krsek, T. Pajdla, and V. Hlavac. Differential Invariants as the Base of Triangulated Surface Registration. *CVIU*, 87:27–38, 2002.
- [11] M. L. Leslie Ikemoto, Natasha Gelfand. A Hierarchical Method for Aligning Warped Meshes. *Proc. of 3DIM*, pp. 434–441, 2003.
- [12] P. J. Rousseeuw. Least Median of Square Regression. *J. American Stat. Assoc.*, 79:871–880, 1984.
- [13] R. Sagawa and K. Ikeuchi. Taking Consensus of Signed Distance Field for Complementing Unobservable Surface. *Proc. of International Symposium on the CREST Digital Archiving Project*, pp. 127–138, 2003.
- [14] J. A. Schouten. *Tensor Analysis, 2nd. ed.* Dover Pub. Inc., 1989.
- [15] G. C. Sharp, S. W. Lee, and D. K. Wehe. ICP Registration Using Invariant Features. *IEEE Trans. on PAMI*, 24(1):90–102, 2002.
- [16] G. Turk and M. Levoy. Zipped Polygon Meshes from Range Images. *ACM SIGGRAPH Computer Graphics*, pp. 311–318, 1994.
- [17] Z. Zhang. Iterative Point Matching for Registration of Free-Form Curves and Surfaces. *IJCV*, 13(2):119–152, 1994.