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3D Shape Registration with Estimating Illumination and Photometric Properties of a Convex Object

Lukáš Cerman[1], Akihiro Sugimoto[2], and Ikuko Shimizu[3]

[1] Czech Technical University, Faculty of Electrical Engineering, Center for Machine Perception
121 35 Prague 2, Karlovo naměstí 13, Czech Republic
cermal1@fel.cvut.cz

[2] National Institute of Informatics
2-1-2 Hitotsubashi, Chiyoda-Ku Tokyo 101-8430, Japan
sugimoto@nii.ac.jp

[3] Tokyo University of Agriculture and Technology, Japan
ikuko@cc.tuat.ac.jp

Abstract *To build a full 3D model of a physical object, multiple partially overlapping parts of an object model need to be merged. Since modern range finder devices provide color information in addition to the depth information, the color can be used to help the registration process as well as to qualitatively improve the reconstructed model by incorporating radiometric properties of its surface to the model description. We propose a novel method for a convex Lambertian object which enables us to perform 3D registration and, at the same time, to estimate radiometric properties of the object. This is achieved in an iterative manner by extending the Iterative Closest Point (ICP) algorithm by adding steps recovering incident illumination and surface albedo. The iterative process is repeated until convergence similarly as in the traditional ICP algorithm. The use of albedo in shape registration improves convergence of the ICP algorithm and reduces ambiguities caused by rotationally symmetric objects.*

1 Introduction

Detailed models describing physical objects are used in many areas such as in object recognition, quality control, film or computer games industry. If the object of interest was not designed using *computer aided design* software (CAD) or there is no access to original CAD model, it is necessary to obtain the model directly from the physical object.

Though it is possible to create the detailed 3D model using some modeling software, it can be very labor intensive to realize. Thus automating the whole modeling process has attracted a substantial interest in recent years.

A typical device for 3D shape acquisition – *a range finder* – cannot reconstruct a full model of an object in a single step, it can capture only a part of the object. Thus the object of interest has to be captured many times – each time from different viewpoint. Partial models acquired in individual scanning steps have to be merged to form a full model of the object.

In many applications, not only shape but also the radiometric properties of the object surface are required. Since modern range finder devices can capture both depth and color information at the same time, this task can be accomplished while modeling shape. Color properties of the object shape may also help to merge partial models to a full model.

2 Problem Specification

In this work, we focus on reconstruction of a full 3D model of a convex Lambertian object. Together with shape, we aim to reconstruct the radiometric properties of the real world object. Since we assume Lambertian surface, the radiometric properties correspond to albedo. We assume the complex illumination is known up to a rotational transformation relating illuminant coordinate system to the coordinate system attached to a range finder device. The illumination distribution, which is relatively distant, can include an arbitrary combination of a point source, an extended source and diffuse light. The convex shape of the object ensures that there are no cast shadows or interreflections.

Since the object shape is captured in multiple steps – each from different viewpoint – multiple overlapping 3D shapes are captured. Each 3D shape describes a slightly different part of the object. There is an unknown 3D Euclidean transformation consisting of a rotation and translation which brings each pair of overlapping 3D shapes into a correspondence. Finding this transformation is called *registration* of 3D shapes. We say that the shapes are well registered when they are geometrically close and their colors match.

The task of 3D shape registration is usually formulated as an optimization problem. A cost function is based on a metric estimating the distance between the overlapping parts of the registered shapes. The optimization problem is non-convex in general. Various methods have been proposed in the computer vision literature to solve it. Obviously, registration of two 3D shapes can only be achieved if there is a common surface area on both shapes. If the correspondences between points in different images were known a priori then the solution to registration would be obtained directly. In practice, the transformation registering both surfaces has to be estimated indirectly from corresponding points on both surfaces. The method described in this paper utilizes geometric as well as the radiometric properties of both shapes to assist the process identifying common points.

To solve a non-convex optimization problem, we assume that the transformations coupling all 3D shapes pairs are roughly known. This prevents local optimization method from getting stuck in some local minima far from the optimal position. The problem of estimating rough registration has been addressed – among others – in works of Krsek et al. [11, 10], Wyngaerd [23] or Sara et al. [20].

3 Related Work

As our work incorporates recent results on illumination analysis to a 3D shape registration process we divided this section in two separate parts. In Section 3.1 we summarize state of the art methods for 3D shape registration, particularly we concern on ICP algorithm which has become most popular method used to register 3D shapes. In Section 3.2 we provide a brief overview of current state of the illumination analysis in computer vision.

3.1 3D Shape Registration

Registration of 3D shapes – especially with only a partial overlap – is a difficult task. In 1992, Besl and McKay introduced the Iterative Closest Point (ICP) algorithm for rigid registration of two shapes [2]. Independently to Besl and McKay, Chen and Medioni described a similar method for 3D shape registration [3].

The ICP algorithm became very popular and many improved variants of the original approach appeared since its original publication. The basic principle of all ICP versions is a repetition of matching potentially corresponding points on both shapes and minimizing the distance between them. The matching step followed by a minimization step is repeated until a convergence criterion is reached. The individual variants differ in (1) the way they select and (2) match points on the two shapes, (3) how they weight pairs of points or even reject some points, (4) used error metric based on the point pairs, (5) and minimization of the error metric. The ICP variants also differ in the kind of information they use in the registration process – some use only geometric features, others use texture features in addition to geometric features.

3.1.1 Registration with Geometric Features Only Rusinkiewicz and Levoy provided [19] an overview and a comparison of many variants of ICP algorithms. They concentrated mainly to effectiveness and speed, not to robustness. They showed that a random point sampling based on the surface normal distribution provides more robust results than a uniform sampling. Further, selecting point pairs by projecting points from one shape on the other one results in faster but less robust approach. Weighting of corresponding pairs has only a small effect on the ICP speed or robustness and is data-dependent. Rejection of potential outliers influences more the robustness than the convergence speed. According to the investigation, they proposed a new efficient ICP variant suitable for a real-time application.

The original ICP cannot be used in general to register shapes with only partial overlap, which is the case of partial 3D scans we deal with. To make registration of partly overlapping shapes more robust, Turk [21] proposed a strategy rejecting pairs that include points on shape boundaries. Since the cost of forcing this constraint is usually low and in most cases its use has few drawbacks, Rusinkiewicz and Levoy proposed to always use this strategy.

Pajdla and Van Gool [14] proposed the Iterative Closest Reciprocal Point (ICRP) algorithm that enforces symmetry on similar/closest point selection – given a point $p \in P$ on first surface P and the closest point $m \in M$ on the second surface M, m is backprojected on P by finding closest point $p' \in P$. If $|p - p'| > \epsilon$, the pair is rejected. Such modification improved robustness of the original ICP algorithm. However, the monotonic convergence is not guaranteed.

Chetverikov et al. [4] proposed the Trimmed Iterative Closest Point (TrICP). The algorithm is based on the use of Least Trimmed Square (LTS) approach. LTS [18] means sorting the squared errors in an increasing order and minimizing the sum of a certain number of smaller values. TrICP provides results comparable to the ICRP algorithm when it starts from a good initial position and performs better when the initial position is not properly established. Unlike ICRP, TrICP preserves the proven convergence [4] of the original ICP. An alternative approach to robustify the ICP is to use Least Median of Square estimator (LMS), which was proposed by Masuda et al. [12]. However, LMS cannot be implemented to ICP without breaking its guaranteed convergence and, as shown in [4], LTS has better statistical efficiency and smoother objective function which is less sensitive to local defects.

Okatani and Sugimoto [13] classified a surface surrounding each point on the shape to one of eight classes based on the local curvature properties. Corresponding pairs containing points from different classes were rejected. Further, to reduce risk of getting stuck in a local minima, the objective function describing the distance between the corresponding points was extended by adding a skewness constraint. This constraint ensures that the direction and the magnitude of point movement caused by the transformation is becomes as uniform as possible among neighboring points.

3.1.2 Registration with Texture Features Weik [22] proposed an algorithm that uses texture data in seeking for corresponding point pairs. Given a point $p \in P$ on the shape P, a point $m_c \in M$ on the shape M is found by projecting point p onto M. The texture intensity gradient at m_c is used to predict a position of a point $m \in M$ with the texture intensity most similar to p. This point is used to form a corresponding pair (p, m).

Johnson and Kang [9] proposed an algorithm for registration of textured 3D shapes. They extended the original ICP distance metric by incorporating a color similarity term. The closest points are sought in 6D space – three spatial dimensions and three color dimensions. Given a set of corresponding point pairs, the Euclidean transformation is found by minimizing the least square distance of the spatial coordinates only.

Godin et al. [7] proposed another method that uses texture data in shape registration. Moreover, they generalized the idea to use not only texture but arbitrary attributes of a point on the shape – e.g., curvature properties. Attributes play two roles in the proposed method. First, their distribution orients the random sampling of shape points. Second, the determination of the closest point is constrained by the attribute compatibility. To robustify search for corresponding points, the random sampling is biased to favor points that are more likely to appear in both images. This extension increases the possibility they belong to an overlapping area in both images. The sampling is driven by a probability density function created as an intersection of attribute histograms of both images.

Pulli et al. [15] described a new method that is not derived from the ICP algorithm. Unlike ICP, the proposed method does not divide optimization in two steps. They proposed to directly minimize one objective function that takes into account the distance between the registered shapes. In their method, a virtual camera is rigidly attached to each surface. One surface is then projected on the image plane of the second camera and then compared with the image of the second surface. The images are compared with respect to range, color and silhouette properties. Correct transformation relating both surfaces is found by minimizing the distance of the projected images.

3.2 Estimating Radiometric Properties of 3D Shapes

Basri, Jacobs [1] and Ramamoorthi, Hanrahan [17] showed independently that the arbitrary complex, distant illumination can be described in a frequency domain using *spherical harmonics*. They showed that modeling illumination of a convex Lambertian object is analogous to the convolution of the lighting function using a kernel that represents Lambertian reflection. This kernel acts as a low-pass filter with 99.2 percent of its energy in the first nine components – the zero, first and second order harmonics. This yields to a compact description of an arbitrary complex illumination.

In their later work Ramamoorthi and Hanrahan [17] demonstrated the use of spherical harmonics description in lighting simulation. They showed that global illumination can be expressed in a very compact form using just a 4-by-4 matrix multiplication.

Du et al. [6] used the spherical harmonics description in recovering properties of a moving Lambertian object under unknown, complex illumination. Using geometry reconstructed by a Shape-From-Motion method, they recovered albedo and the illumination distribution.

4 Our approach

The proposed method consists of two steps. The two steps are repeated until the geometrical distance of the registered shapes reaches its minima.

In the first step, we fix the illumination coefficients assuming the illumination is known and minimize geometrical distance of the 3D shape pair using traditional ICP steps. The fixed illumination is used to derive albedo from shape normals and recorded surface brightness. Estimated albedo of surface points helps in finding good point correspondences between the shapes.

In the second step, we fix the geometry assuming the relative shape positions are known. We use shape points viewed in different poses to estimate illumination. More precisely, we assume that the complex illumination is already estimated and we seek a rotational transformation relating global illumination to the coordinate system of the range scanner.

4.1 Estimating geometry

As a base of our geometry estimation step, we use the Trimmed Iterative Closest Point (TrICP) method described by Chetverikov et al. [4]. The TrICP method allows to register only partially overlapping shapes while preserving guaranteed convergence of original ICP [2]. TrICP consists of the following four basic steps:

1. *matching step* – pair each point $p \in P$ to its closest point $m \in M$,
2. *filtering step* – compute the mean square error (MSE) between the paired points, and keep only a certain number of point pairs having the smallest squared errors,
3. *optimizing step* – compute the transformation minimizing MSE between the remaining point pairs,
4. *update step* – apply the optimal transformation to P and update MSE.

Our *matching step* is accomplished in two sub-steps. First, for each point $p \in P$, the closest point $m_c \in M$ is found using an Euclidean metric. Then, using point normals and estimated illumination, an albedo of both points p and m_c is estimated. If the albedo is similar enough then point m_c is accepted as a corresponding pair (p, m_c). On the other hand, if albedo of points p and m_c differ then the neighborhood of the point m_c is sought to find a point $m \in M$ which is compatible with point p. If such a point is found then a corresponding pair (p, m) is created.

Next, this matching is repeated in reciprocal direction – matching points from shape M to points at shape P.

At the beginning of our *filtering step*, all point pairs that contain points on the shape boundary are rejected. Since the cost of forcing this constraint is usually low and in most cases its use has few drawbacks. We note that Rusinkiewicz and Levoy [19] concluded that this is an efficient way to remove outliers.

The remaining points are then sorted by their squared error – a squared Euclidean distance between the points in a pair. Only a certain number of the least distant points is kept. This step should remove remaining outliers and robustify the whole registration process.

To compute the transformation minimizing MSE among remaining point pairs in our *optimizing step*, we use an optimization method proposed by Horn [8]. This method gives a closed-form solution for estimating Euclidean transform from a corresponding point pair set using quaternions.

4.2 Estimating Illumination

Under normal conditions, light coming from all directions illuminates an object. Reflection of an incident light by a convex Lambertian surface can be described in frequency domain using spherical harmonics [1, 17, 16]. Amount of light reflected by a point on a surface is a function of the surface normal, harmonic coefficients of illumination distribution and the surface albedo. Assuming the illumination, the object surface and the range finder position do not change during scanning process, we can form a set of equations. Each equation relates light reflected by one surface point viewed in two different poses. Having sufficiently many corresponding points, we can recover illumination rotation and surface albedo from the pair of textured 3D shapes.

The global illumination is often represented by a light probe. The light probe is an omnidirectional image recording illumination at a particular point in space. Light probes were proposed by Debevec [5]. Sometimes we use the term light probe instead of the term global illumination in the following explanation.

4.2.1 Spherical harmonics representation Let L denote the distant illumination distribution. The irradiance E is then a function of the surface normal n only and is given by the integral over the upper hemisphere $\Omega(n)$,

$$E(n) = \int_{\Omega(n)} L(\omega)(n \cdot \omega)\, \mathrm{d}\omega\, . \quad (1)$$

Since n and ω are unit vectors, E and L can be parameterized by direction (θ, ϕ) on the unit sphere.

The irradiance E is then scaled by the albedo ρ at the surface point p yielding radiosity B, which corresponds directly to the image intensity.

$$B(p, n) = \rho(p)E(n) \quad (2)$$

As proposed in [1, 17, 16], the illumination distribution L can be expressed in a spherical harmonic representation,

$$L(\theta, \phi) = \sum_{l,m} L_{lm} Y_{lm}(\theta, \phi)\, , \quad (3)$$

where $Y_{lm}(\theta, \phi)$ are spherical harmonics basis and L_{lm} are spherical harmonics coefficients.

The irradiance E can be then expressed in following form

$$E(\theta, \phi) = \sum_{l,m} \hat{A}_l L_{lm} Y_{lm}(\theta, \phi)\, , \quad (4)$$

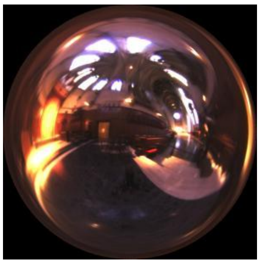
where coefficients $\hat{A}_l$ are

$$\hat{A}_l = \begin{cases} \frac{2\pi}{3} & \text{for } l = 1 \\ 0 & \text{for } l > 1, \text{odd} \\ 2\pi \frac{(-1)^{\frac{l}{2}-1}}{(l+2)(l-1)} \left[\frac{l!}{2^l (\frac{l}{2}!)^2}\right] & \text{for } l \text{ even} \end{cases} . \quad (5)$$

As noted in [17], $\hat{A}_l$ decays so fast that we need only first coefficients of order $l \leq 2$. Equivalently, the irradiance is well approximated by only 9 parameters.

The nine illumination coefficients can be computed from measured illumination by integrating against the spherical harmonics functions. Each color channel is treated separately.

$$L_{lm} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} L_{lm(\theta,\phi)} Y_{lm}(\theta, \phi) \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi \quad (6)$$



$L_{l,m}$	Red	Green	Blue
$L_{0,0}$	0.078908	0.043710	0.054161
$L_{1,-1}$	0.039499	0.034989	0.060488
$L_{1,0}$	-0.033974	-0.018236	-0.026940
$L_{1,1}$	-0.029213	-0.005562	0.000944
$L_{2,-2}$	-0.011141	-0.005090	-0.012231
$L_{2,-1}$	-0.026240	-0.022401	-0.047479
$L_{2,0}$	-0.015570	-0.009471	-0.014733
$L_{2,1}$	0.056014	0.021444	0.013915
$L_{2,2}$	0.021205	-0.005432	-0.030374

Figure 1: Light probe image of Grace Cathedral we used in our experiments and its first nine spherical harmonics coefficients.

Since we are considering only $l \leq 2$, the irradiance is simply a quadratic polynomial of the coordinates of the normalized surface normal. With setting $n = (x\ y\ z\ 1)$, we can rewrite the irradiance equation in the form

$$E(n) = n^T M n\, , \quad (7)$$

where M is a symmetric 4-by-4 matrix. Each color channel has its own matrix M. The matrix M is obtained by expanding equation (4),

$$M = \begin{pmatrix} c_1 L_{22} & c_1 L_{2-2} & c_1 L_{21} & c_2 L_{11} \\ c_1 L_{2-2} & -c_1 L_{22} & c_1 L_{2-1} & c_2 L_{1-1} \\ c_1 L_{21} & c_1 L_{2-1} & c_3 L_{20} & c_2 L_{10} \\ c_2 L_{11} & c_2 L_{1-1} & c_2 L_{10} & c_4 L_{00} - c_5 L_{20} \end{pmatrix}, \quad (8)$$

where L_{lm} are first nine coefficients of the spherical harmonics representation and

$$c_1 = 0.429043 \quad c_2 = 0.511664$$

$$c_3 = 0.743125 \quad c_4 = 0.886227 \quad c_5 = 0.247708\, .$$

To rotate the illumination, we can simply apply inverse rotation to the normal n. Thus irradiance induced by the illumination rotated by R^{-1} can be expressed using

$$E(n) = n^T R^T M R n\, , \quad (9)$$

or

$$E(n) = n^T M' n\, , \quad (10)$$

where $M' = R^T M R$ is a rotated light probe expressed in terms of spherical harmonics.

4.2.2 Estimating light probe rotation In this step, we assume geometry is fixed and correctly registered. For each point $p \in P$ its closest point $m \in M$ is found using an Eucledian metric. Measured brightness of points p and m can be expressed using

$$\begin{aligned} B(p) &= \rho_p n_p^T M n_p \\ B(m) &= \rho_m n_m^T M n_m \, . \end{aligned} \tag{11}$$

Since both points p and m represent the same point on the 3D shape viewed in a different pose, albedo of those points must be equal – $\rho_p = \rho_m$. Thus, we can rewrite (11) in

$$B(p) n_m^T M n_m = B(m) n_p^T M n_p \, . \tag{12}$$

Having sufficiently many corresponding points p and m we can solve linear equation system (12) for unknown symmetric matrix M.

Since we assume we already know illumination up to the unknown rotation, we need to find the rotation R transforming measured light probe matrix L to the light probe matrix M estimated in the previous step. By solving linear system

$$LR' - R'M_c = 0 \, , \tag{13}$$

where $c \in r, g, b$ represents three color channels, we obtain transformation matrix R'. We use singular value decomposition

$$R' = USV^T \tag{14}$$

to enforce that the estimated transformation R is a rotation,

$$R = UV^T \, . \tag{15}$$

5 Experiments

To simulate a range finder device, we created synthetic objects using a 3D modeling software. We used the 3D Studio Max to render depth-map and texture images from different view-points. To have better control over the illumination and not to rely on the shading algorithms in 3D Studio Max, the texture images were rendered with uniform illumination. Since the texture does not contain any shading effects, it can be used as a representation of the surface albedo. The effect of incident illumination was simulated later in our software. We applied illumination with the light probe shown in Figure 1 to every mesh point according the equation (7).

To see the effectiveness of the proposed method, we evaluated its ability of establishing the correct light probe rotation as well as the object shape registration. To show the advantage of using surface albedo properties in the shape registration, we registered a rotationally symmetric object, see Figure 2. Both registered shapes have roughly 60 000 mesh points. To obtain the best results, we used all mesh points, although it might be possible to use some decimated mesh in order to reduce the computational time. Before running our algorithm, we manually established rough pre-alignment of both shapes by selecting a few corresponding points with hand and minimizing the squared distance between them. The TrICP culling rate was set to 30% so that 70% of best correspondences remained in each iteration.

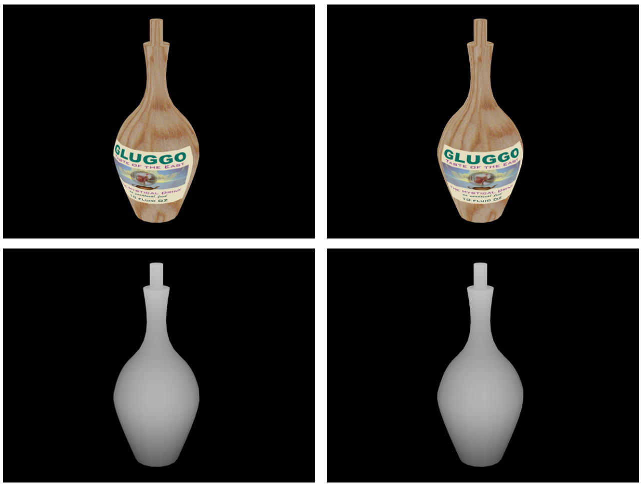


Figure 2: Rendered images used to simulate the range finder output. Images on the right side shows the object rotated by 20 degrees. Upper row shows the albedo map (without illumination applied), bottom row shows the rendered depth-map.

Figure 3: (left) shape of the rotationally symmetric object registered without the use of albedo properties, (right) the same shape registered using the albedo properties.

In the illumination estimating step, we simulated perfect texture data by taking albedo value from the first point in each correspondence pair, applying illumination to it according the normal of the second point in the pair and storing the shaded value as the texture of the second point in correspondence pair. This enables us to control the quality of the texture. In each iteration we used LTS approach to filter out the outliers and robustify our solution. First, we kept only 70% pairs with smallest Euclidean distance. Second, we analyzed normals mismatch among all pairs and kept only 50% pairs with most similar normal direction.

It was sufficient to run just three iterations of our algorithm to reach convergence. In the experiments, we used identity as the true light probe rotation. The estimated probe rotation found by our algorithm was

$$\begin{pmatrix} 1.00015 & -0.000764272 & -0.00038349 & 0 \\ -0.00076271 & 1.004 & 0.00205964 & 0 \\ 0.00151472 & -0.00807858 & 0.995859 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} . \tag{16}$$

It can be concluded, that the estimate is very close to the ground truth. Figure 3 shows the result of a registration of the rotational symmetric object. Image on the left is the result of registration without the use of albedo properties, image on the right is the result of our method, which utilizes albedo to help the process of finding good point correspondences. We note that both registrations were started from the same initial position.

We also experimented with a noisy data to test the robustness of the proposed method. We added normally distributed noise with mean zero to the texture value of each mesh point and tested the convergence of our algorithm. In each test, we added noise with a different standard deviation. Similarly, we added noise to the surface normal of each mesh point – we rotated each normal by the random angles, in each direction X, Y, Z independently. We used normally distributed noise with mean zero and, in each experiment, we used different value of standard deviation. It showed up, that the the algorithm diverges when the value of standard deviation of the texture noise exceeds 0.001 and standard deviation of the normal noise exceeds 0.01.

6 Conclusions

We proposed a novel method that enables us to build a 3D model of a convex Lambertian object. We showed that it is possible to estimate the shape registration together with estimating of the illumination and photometric properties of the object surface. We showed that using albedo information in the registration process can help to resolve ambiguities caused by rotationally symmetric objects. The proposed method fits well in the traditional ICP scheme, thus it can be easily incorporated in many existing applications.

7 Future Work

We observed that the proposed method might be sensitive to noise in normal directions as well as the noise in albedo values. The method should be further examined to find out whether the problem could be ill conditioned. In that case, some stabilization method needs to be found.

The future work can also aim at improving the seek for compatible corresponding points in the geometry optimization step. Current version simply compares albedo values and does not incorporate the point neighborhood or some higher level features.

To robustify estimation of light probe rotation only points from areas with uniform normal distribution and albedo properties should be used. This step can be accomplished by filtering correspondences using some criterion before solving linear system (12). Another robustification may be reached by implementing some closed form solution to estimate a probe rotation instead of finding general transformation and enforcing the rotation using SVD.

To verify the applicability of the proposed method under a real conditions, the method should be tested on the real data from a range finder device.

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